State Estimation of a Target Measurements using Kalman Filter in a Missile Homing Loop

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Abstract: The main function of a missile seeker subsystem also known as homing eye is to provide target measurements such as line of sight (LOS) rate and closing velocity that are required to mechanize the guidance law. This geometric LOS is prone to be corrupted by noise and the acceleration of the interceptor cannot be relaxed in stressing engagement situations as these acceleration commands of the interceptor are issued perpendicularly to the instantaneous missile target LOS that are proportional to LOS rate and closing velocity. With these thoughts, this paper is concerned with the extraction of LOS rate from its raw form using an optimal Kalman filter. Therefore, in this context LOS rate reconstruction process in a missile homing loop has been treated extensively in this paper. The key challenges associated with the effective design of homing guidance system are also discussed in this paper. The performance evaluation of the fixed gain filter and Kalman filter for estimation of LOS rate and target maneuver has been given more emphasis in this paper. Finally, the implications in evaluating the filter performance shows that the Kalman filter can give substantial improvement and provide the best optimal solution for state estimation in a missile homing loop.

Keywords: Missile seeker, Proportional Navigation, Line of sight, fixed gain filter and Kalman filter.

I. Introduction

Guidance is the means of by which a missile steers, or is steered, to a target. A guided missile is guided according to a certain guidance law, and homing guidance systems perform such a task. A meaningful comparison of homing guidance system for missiles requires realistic models for the missile and its target engagement geometry model in order to accurately evaluate for terminal miss distance. This model should include the important dynamics and system nonlinearities that influence performance, and yet be representative of missile in general. In the simplest form, the principal elements that make up a missile guidance system are illustrated in Fig.1.

For a missile, the inputs are target location and missile-to-target separation. The desired output is that the missile has the same location as the target. The missile does this by using a certain guidance system and flying according to a certain guidance law. [1]



Fig. 1 Principal Elements of a Missile Guidance System. Fig.2Traditional Guidance, Navigation and Control Topology

II. Missile Homing Loop

The traditional guidance, Navigation and control topology [2] for a guided missile comprises guidance filter, guidance law, autopilot and inertial navigation components. Each component may be synthesized by using a variety of techniques; all the components are connected to form a homing loop, which is shown in Fig.2. Kalman filter serve as an optimal digital noise filter in a missile homing loop. Also the missile flight control system is one element of the overall homing loop Fig.3 is a simplified blockdiagram of the missile homing loop configured for the terminal phase of flight when the missile is approaching intercept with the target. The terminal sensor, typically an RF or IR seeker, measures the angle between the missile-to-target Line-of-sight vector and an inertial reference which is called the LOS angle



Fig.3 Information flow in Missile Guidance Loop

In the missile homing loop described in Fig.3 the state estimator e.g a Kalman filter, uses LOS angle measurements to estimate LOS angle rate and perhaps other quantities such as target acceleration. The state estimates feed a guidance law that develops the flight control commands required to intercept a target. The flight control system forces the missile to track the guidance commands, resulting in the achieved missile motion. The achieved missile motion alerts the relative geometry, which then is sensed and used to determine the next set of flight control commands, and so on. This loop continues to operate until the missile intercepts the target. Therefore in the parlance of feedback control, the Homing loop is a feedback control system that regulates the LOS angle rate to Zero. As such, the overall stability and performance of the control system are determined by the dynamics of each element in the loop [3].

Therefore as stated earlier, the state estimator forms an estimate of the LOS angle rate, which in turn is input to the guidance law and, also the same estimator can also estimate the other quantity such as target acceleration, therefore from the theory if homing loop it was understand that state estimator for estimating LOS rate and target acceleration plays a vital role resulting in the achieved missile motion. With these insights the paper focuses on state estimator for overall homing loop performance requirements in the presence of target maneuver and other disturbances in the system example terminal sensor which can negatively impact missile performance. The remainder of this paper is divided into some background information given in section 3, Source of miss distance in section 4, two dimensional missile target planar geometry in section 5, Kalman filter implementation in section 6, experimental results and analysis in section 7.

III. Related Work

[4] This paper presents how Kalman filter can be integrated in a homing loop. The implementation of missile guidance laws on collision course trajectories with bearings only measurement may result in bad range observeability. This in turn may result in poor engagement performance for these guidance algorithms that explicit the range information in their steering law. Maneuvering away from the collision course trajectory improves range observeability. To propose a sizing criterion for the maneuver, this work presents a new guidance strategy that exploits the information from the error covariance matrix if the homing loop integrated Kalman filter in the framework of a pursuit evasion game. The new strategy has been tested in several scenarios against other guidance laws (such as the) and Monte Carlo simulation shows that then proposed solution is able to improve engagement performance in terms of both miss distance and range observability.

[5] This paper explains a two-step optimal estimator (TSE) as a novel estimation methodology to the general non-linear three-dimensional problem of tracking a maneuver target. It is an attractive alternative to the standard extended Kalman filter. The target tracking performance of the TSE is shown to be better than an EKF implemented in either inertial or modified spherical coordinates. In the passive case, where bearing/elevation angles only are measured, the TSE yields excellent range and target acceleration estimates. In the active case, where range measurement is available as well, a homing missile employing closed loop optimal guidance based in TSE state estimator obtains smaller miss distance than with EKF.

[6]This paper presents an application of the Kalman filter algorithm to maneuvering targets state estimation from line of sight and range/range-rate measurements. These target state estimates are used by the homing loop (guidance law) to guide the interceptor towards the target. Standard terminal homing guidance laws such as Proportional Navigation have proven to be very effective against non-maneuvering targets. Modern guidance laws have been developed to try to improve terminal homing (miss distance) performance in the presence of maneuvering targets. However these new guidance laws required precise target state estimation and robustness, accuracy of the standard target state estimators to target maneuvers have proven to be one of the limiting factors on improving the interceptor's terminal performance Kalman filter as an estimator was able to get good estimates of targets velocity and acceleration improving homing performance.

The problem of tactical missile guidance is very challenging and has been treated using several methodologies in the past four decades. Major techniques can be grouped under classical guidance laws, modern guidance laws based on intelligent central methods. Each technique has some advantages and disadvantages. While implementing in a practical system guidance law selection is dictated by nature of flight profile like boost,

midcourse, terminal homing etc., and also miss distance and a single-shot kill probability. [7] A sample structure in information flow in missile guidance loop is shown in Fig.3

IV. Source of Miss Distance

The first problem is to maximize the single-shot kill probability [SSKP] by minimizing the miss distance. The sources of miss distance will occur from initial heading error, acceleration bias, gyro drifts (if gyros are used in seeker stabilization) glint (scintillation noise), Receiver noise, fading noise and angle noise (due to varying refraction with frequency diversity). The second problem is to preserve the stability of parasitic attitude loop. The third problem is filtering must be done to limit noise perturbations of the seeker and actuators, so that power consumption, saturation, g-limiting etc., will not be excessive. A successful guidance design requires a compromise that meets all three major problems.

V. Missile Target Planar Geometry

The formulation of the planar intercept (pursuit-evasion) has been given in Fig.4 which illustrates the planar (two-dimensional) engagement geometry and defines the angular and Cartesian quantities depicted their in.



Fig.4 Planar Engagement Geometry

In the Fig.4 the x-axis represents downrange, while the y/z axis can represent either cross range or altitude respectively. For simplicity, a flat earth model with an inertial coordinate system fixed to the surface of the earth is assumed. The positions of the missile (M) and target (T) are shown with respect to origin (O) of the coordinate system, $\overline{r_M}$ and $\overline{r_T}$ respectively thus the relative position vector or line-of-sight vector is given by $\overline{r} = \overline{r_T} - \overline{r_M}$, from the Fig the line-of-sight angle can be found using trigonometry in terms of relative separation components as

$$\boldsymbol{\lambda} = \tan^{-1} \left[\frac{R_2}{R_1} \right] \tag{1}$$

Also the relative velocity vector is given by $\dot{\bar{r}} \triangleq \bar{V} = \overline{V_T} - \overline{V_M}$ (2)

And if we define the relative velocity components to
$$beV_{TM1}$$
 and V_{TM2} , the line-of-sight rate can be (differentiated) obtained by direct differentiation of (1) and after some algebra the expression for the LOS rate to be

$$\dot{\lambda} = \frac{R_1 \cdot V_{TM2} - R_2 \cdot V_{TM1}}{R^2}$$
(3)

The above equation is the measurement model of the actual Line-of-sight rate measured by missile seeker for state estimate observational update.

VI. Kalman Filter Performance in Homing Loop

Theoretically the Kalman filter is an estimate for what is called the linear quadratic problem, which is the problem of estimating the instantaneous "state" of a linear dynamic system perturbed by white noise – by using the measurements linearly related to the state but corrupted by white noise. Kalman filter gives the best optimal solution for the linear state estimation problem considering noise in its model as White Gaussian [8]

Kalman filter act as an optimal digital noise filters in the missile homing loop in an attempt to estimate relative position, relative velocity and target acceleration.

A. Discrete Kalman Filter Equations

To apply Kalman filtering theory, the model of the real world must be represented by a system dynamic model of the form

 $\dot{X} = FX + GU + W \text{ or}(4)$

 $x_k = \Phi_{K-1} \cdot X_{k-1} + W_{k-1}$ Where $W_k \sim N(0, Q_k)$ (5)

To represent the model in state space form, the X is a column vector describing the state of the system is taken as one-dimensional third order of the form

$$\mathbf{X} = \begin{bmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\dot{\lambda}} \\ \boldsymbol{\bar{a}}_T \end{bmatrix}$$
(6)

Also from the Eq. 5 the fundamental or state transition matrix in discrete form is

(8)

$$\Phi_k = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$
(7)

And the measurement model is given by the equation

 $Z_k = H_k \cdot X_k + V_k$

 $V_k \sim \mathcal{N}\left(0, R_k\right)(9)$

From the Eq. 8 the measurement sensitivity matrix defining the linear relationship between state of the dynamic system and measurement that can be made is taken as $H_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$. Here V_k is the measurement noise and in this numerical example the angular measurement noise is taken as 1 milliradian (1mr). The state estimate extrapolation is to predict the next state and is given by

 $\hat{X}_{k}(-) = \phi_{k-1}\hat{X}_{k-1}(+)$ (10) And the error covariance extrapolation is given by $P_{k}(-) = \phi_{k-1}P_{k-1}(+)\phi_{k-1}^{T} + Q_{k-1}$ (11) where the initial value for the prediction covariance is $\begin{bmatrix} \sigma^{2} \text{ ration} & 0 & 0 \end{bmatrix}$

$$P_{k-1} = \begin{bmatrix} 0 & noise & 0 \\ 0 & \left[\frac{V_{M}.HE}{57.3}\right]^{2} & 0 \\ 0 & 0 & n^{2}_{T} \end{bmatrix} (12)$$

The state estimate observational and error covariance update is given by

$$\hat{X}_{k}(+) = \hat{X}_{k}(-) + \bar{K}_{k}[Z_{k} - H_{k}\hat{X}_{k}(-)]$$
(13)

$$P_{k}(+) = [1 - \bar{K}_{k}H_{k}]P_{k}(-)$$
(14)
Finally the Kalman Gain Matrix is given by

$$\bar{K}_{k} = P_{k}(-)H_{k}^{T}[H_{k}P_{k}(-)H_{k}^{T} + R_{k}]^{-1}$$
(15)

VII. Experimental Results

In order to demonstrate the working of Kalman filter in homing loop consider all the discrete Kalman filter equations described in section 6 with initial conditions. The Kalman filter recursion process is shown in Fig. 5.



Fig. 5 Kalman Filter Recursion

In this simulation the closing velocity, acceleration of the target and missile velocity magnitude is assumed to be 400 m/s, 96.6 m/s² and 3000 m/s². In order to evaluate the performance of Kalman filter as a part of missile homing loop, the line of sight (LOS) rate and 3G target maneuver estimation with Kalman filter has to be compared with that of fixed gain filter which was the previous analysis of this paper. With the same initial conditions the LOS and LOS rate estimation made with two state fixed gain filters along with residual error are

shown in Fig. 6, Fig. 7, Fig. 8 and Fig. 9. These figures compares the actual LOS rate to the filter estimate of the LOS rate measurement with 1 Mr of noise, navigation ratio of 3, sampling time of 0.1 s and ' β ' for two values such as 0.3 and 0.8. The Fig. 6 shows decreasing β effectively increases filters bandwidth and filter no longer lags the actual LOS rate when β is reduced from 0.8 to 0.3, however decreasing β increases the noise transmission of the filter. Also the Fig. 7 shows increasing β tends to decrease the bandwidth of the filter and LOS rate is smooth but lags actual LOS rate indicating filtering is sluggish. Also





With the same initial conditions the LOS and LOS rate estimation made with three state fixed gain filters along with residual error are shown in Fig. 10, Fig. 11, Fig. 12 and Fig. 13.







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On the other hand Kalman filter recursion though they are close to the fixed gain filters, they can give substantial performance with the propagation of its covariance matrix P_k and a dynamically computed gain matrix K_k . Now the Fig. 14 and Fig. 16 shows the LOS rate estimation using Kalman filter which shows the filter is neither sluggish nor noisy like fixed gain filters. The simulation run given in Fig. 14 and Fig. 16 with angular measurement noise V_k of 1 Mr and 10 Mr shows the estimated LOS rate is perfectly following the actual LOS rate with no or very less residual.







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500

1000

1500

Fig. 17 Residual for 10 Mr of Noise

-0.06

2000

2500

3000

3500

S.No	Time of	Actual LOS Rate(Rad/s)		Estimated I	LOS Rate(Rad/s)	Residual(Rad/s)		
	Flight(s)	β =0.3	β =0.8	β =0.3	β =0.8	β =0.3	β =0.8	
1	0	0.00000670	0.00000670	-0.00359	-0.00049	-0.00073	-0.00122	
2	500	0.00335	0.00335	0.01132	0.00364	0.00164	0.00094	
3	1000	0.00670	0.00671	-0.00905	0.00665	-0.00159	-0.00190	
4	1500	0.01006	0.01006	0.00796	0.00976	-0.00051	-0.00093	
5	2000	0.01341	0.01342	0.00627	0.01306	-0.00171	0.00040	
6	2500	0.01676	0.01677	0.02288	0.01689	0.00134	0.00084	
7	3000	0.02011	0.02013	0.00970	0.01992	-0.00287	-0.00024	
8	3600	0.02650	0.02349	0.03492	0.02368	0.00154	0.00010	

Table1 Readings of L	OS Rate with on	e dimensional	second order	r fixed gair	filter
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For a time of flight of 1 hour, a comparison of actual and estimated LOS rate made by the one dimensional second order filter and one dimensional third order filter for two different values of β and along with the residual error has been tabulated in Table 1 and Table 2.Similarly a comparison of actual and estimated LOS rate made by the Kalman Filter has for 1 Mr and 10 Mr of noise has been tabulated in Table 3.

Table 2. Readings of LOS Rate with one dimensional third order fixed gain filter

S.No	Time of	Actual LOS Rate(Rad/s)		Estimated LOS	Rate(Rad/s)	Residual(Rad/s)	
	Flight(s)	β =0.3	β =0.8	β =0.3	β =0.8	β =0.3	β =0.8
1	0	0.0000067	0.00000067	0.00513	-0.00143	-0.00513	0.00143
2	500	0.00335	0.00335	0.01685	0.00381	-0.01350	-0.00046
3	1000	0.00670	0.00670	-0.00471	0.00604	0.01142	0.00065
4	1500	0.01005	0.01006	-0.00002	0.01037	0.01008	-0.00031
5	2000	0.01341	0.01341	0.01303	0.01351	0.00037	-0.00010
6	2500	0.01676	0.01676	0.02301	0.01628	0.00624	0.00048
7	3000	0.02011	0.02011	0.00665	0.01929	0.01345	0.00082
8	3600	0.02492	0.02304	0.01719	0.02381	0.00773	-0.00040

Table 3. Readings of LOS Rate with Kalman filter

S.No	Time of	Actual LOS Rate(Rad/s)		Estimated LOS Rate(Rad/s)		Residual(Rad/s)	
	Flight(s)	Noise =1Mr	Noise =10Mr	Noise =1Mr	Noise =10Mr	Noise =1Mr	Noise =10Mr
1	0	0.00000067	0.00000067	0.00000094	-0.000000444	-0.0000002	0.00000111
2	500	0.0033564	0.0033539	0.0034053	0.00334482	-0.0000487	0.00000909
3	1000	0.006709	0.00670727	0.0067184	0.00672509	-0.0000088	-0.00001782
4	1500	0.010062	0.0100603	0.0100362	0.01006042	0.0000263	-0.00000005
5	2000	0.013415	0.0134138	0.0133906	0.01340925	0.0000252	0.00000454
6	2500	0.016769	0.0167668	0.0167347	0.01676311	0.0000351	0.000003719
7	3000	0.020118	0.0201198	0.0201414	0.02010458	-0.0000028	0.00001499
8	3600	0.014250	0.0247402	0.0221202	0.02357167	-0.0078778	0.00116863

Another nominal case was run for estimating the target maneuver. Fig. 18 and Fig. 19 shows the estimate of a 3G target maneuver using a one dimensional third order filter with β as 0.8 and sampling time of 0.1 s. From these figures it can be observed that filter takes about 114 to 130 sec with 1 Mr of Noise and 320 to 350sec with 10 mr of Noise atleast to achieve a desired level of target maneuver estimate.







On the other hand, the same estimate of a 3G target maneuver has been performed using Kalman filter with 1 Mr and 10 Mr of angular noise and filter simulation shown in Fig. 21 and Fig. 23. From these figures it can be observed that filter takes about 17 to 20 sec with 1 Mr of Noise and 28 to 30 sec with 10 mr of Noise atleast to achieve a desired level of target maneuver estimate. The residual errors for Kalman filter is shown in Fig. 22 and Fig. 24.





Fig. 24 Residual with 10 Mr of Noise

For a time of flight of 1 hour, a comparison of actual target and estimated 3G Target Maneuver made by the one dimensional third order filter with 1 Mr and 10 Mr of noise along with the residual errors has been tabulated in Table 4. Similarly a comparison of actual target and estimated 3G Target Maneuver made by the Kalman Filter for 1 Mr and 10 Mr of noise has been tabulated in Table 5.

S.No	Time of Flight(s)	Actual Target Maneuver(G)	Estimated Target Acceleration(G)		Residual(G)	
			Noise = 1Mr	Noise = 10Mr	Noise = 1Mr	Noise = 10Mr
1	0	3	-0.05954	0.24050	3.05954	2.75949
2	500	3	2.82393	2.76895	0.17606	0.23104
3	1000	3	3.03795	6.09199	-0.03795	-3.09199
4	1500	3	3.03753	3.39973	-0.03753	-0.39973
5	2000	3	3.07776	3.86386	-0.07776	-0.86386
6	2500	3	2.91854	2.83837	0.08145	0.16162
7	3000	3	3.01521	2.71514	-0.01521	0.28485
8	3600	3	3.00011	3.00026	-0.00011	-0.00026

Table 4. Readings of 3G Target Maneuver made by one dimensional third order fixed gain filter

S.No	Time of Flight(s)	Actual Target	Estimated Target Acceleration(G)		Residual(Rad/s)	
		Maneuver(G)	Noise = 1Mr	Noise = 10Mr	Noise = 1Mr	Noise = 10Mr
1	0	3	0.0000101	0.000004	2.9999	2.99999
2	500	3	3.24236	3.25195	-0.24236	-0.25195
3	1000	3	3.01673	3.17726	-0.01673	-0.17726
4	1500	3	3.14651	3.02455	-0.14651	-0.02455
5	2000	3	3.01093	2.76565	-0.01093	0.23434
6	2500	3	3.08605	2.94013	-0.08605	0.05986
7	3000	3	2.99957	2.75281	0.00042	0.24718
8	3600	3	2.99407	2.89717	0.00592	0.10282

VIII. Conclusion

In this paper it was shown how optimal filters such as fixed gain and Kalman filters could be implemented in a missile homing loop for estimation of LOS rate and target maneuver. It was shown how Kalman filter compared to fixed gain filter can provide the best optimal solution to estimate the LOS rate with no residual or as low as possible. Also it was understood from this paper the Kalman filter can be applied for variety of maneuvering target environments as these filter gains are computed dynamically and accurate measure of covariance matrix.

Nomenclature

 $\begin{array}{l} \gamma_{M} \text{-} Missile \ flight \ path \ angle} \\ \bar{r}_{M} \text{-} Missile \ inertial \ position \ vector} \\ \gamma_{T} \text{-} Target \ flight \ path \ angle} \end{array}$

 λ - LOS angle

- $\dot{\lambda}$ LOS rate
- \bar{r}_{T} -Target inertial position vector
- \overline{V}_{M} Missile velocity vector
- \overline{V}_{T} Target velocity vector
- \bar{a}_{M} Missile acceleration, normal to LOS
- \bar{a}_{T} Target acceleration, normal to VT
- L Lead angle
- r_x Relative position x
- $r_{\rm v}$ Relative position y

 X_k - State vector at time k

 V_k - Measurement Noise

- $f_k/Z_k/h_k$ Non-linear measurement vector
- W_k Process noise
- $\dot{X}(t)/\dot{X}(t)$ -State vector
- W(t)/Y(t)-Input vector
- X_0 -Initial estimate of the state.
- ϕ -State estimation matrix

 $\hat{X}_k(+)$ - Smoothed estimate at time k

- $\hat{X}_{k}(-)$ Predicting the state in $(k-1)^{th}$ interval for k^{th} interval
- β Smoothing coefficient
- T Sampling time

 \overline{K}_{k} - Kalman gain matrix

- Q_k -Covariance matrix of state estimation uncertainty
- R_k Covariance matrix of observational uncertainty
- $P_k(-)$ -Prediction of covariance matrix of state estimation uncertainty
- $P_k(-)$ Smoothing of covariance matrix of state estimation uncertainty
- H_k -Measurement sensitivity matrix

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